Description Logics with Pointwise Circumscription (Extended Abstract)*

Federica Di Stefano[1] , **Magdalena Ortiz**[2] , **Mantas Šimkus**[2]
[1]Institute of Logic and Computation, TU Wien
[2]Department of Computing Science, Umeå University
federica.stefano@tuwien.ac.at, magdalena.ortiz@umu.se, mantas.simkus@umu.se

1 Introduction

Description Logics (DLs) are a family of formalisms for *Knowledge Representation & Reasoning*, specifically designed for describing entities of a problem domain and their relations in the so-called *ontologies* (Baader et al. 2017). Most DLs are based on first-order logic and inherit from it most of its features, including monotonicity. This makes it difficult to capture human-like *common-sense reasoning*, where we may draw default conclusions that can be revised in the light of new information. Several non-monotonic extensions of DLs have been proposed. However, adding non-monotonic features is challenging and often causes an increase in the complexity of reasoning or causes undecidability. A prominent research line here is *circumscribed DLs* (Bonatti, Lutz, and Wolter 2009; Bonatti et al. 2015; Bonatti 2021; Bonatti, Petrova, and Sauro 2022). Circumscription is a powerful tool that was first introduced by McCarthy as an extension of first-order logic. In the basic setting, the intended (or *preferred*) models of a circumscribed theory are its classical models that additionally have minimal extensions of some selected predicates (McCarthy 1980; McCarthy 1986; Lifschitz 1985). In general, additionally to the minimized predicates, one may specify—by means of a *circumscription pattern*—the predicates whose extensions must remain fixed and the predicates that may vary freely during the selection of an intended model.

Circumscribed DLs are an expressive and versatile family of languages, but unfortunately the complexity of reasoning is often very high, and undecidability is easily encountered (Bonatti, Lutz, and Wolter 2009): if roles are only allowed to vary, reasoning in circumscribed $\mathcal{ALCIO}$ is in $\text{NExpTime}^{\text{NP}}$; if roles are minimized or fixed, reasoning is undecidable already in circumscribed $\mathcal{ALC}$. The key reason for the high complexity is the *second-order quantification* that is needed in order to identify the preferred models of a circumscribed DL knowledge base (KB).

Our IJCAI 2023 paper presents an alternative (weaker) notion of circumscription that is useful for knowledge representation and does not use such a powerful second-order quantification, aiming to lower the computational complexity of reasoning. We introduce *pointwise circumscription* in DLs and argue that it yields a useful way to apply a form of the closed-world assumption to DL ontologies and allows to draw intuitive common-sense conclusions from them.

Our pointwise semantics is similar in spirit but orthogonal to the notion of pointwise circumscription introduced by Lifschitz in 1986 for first-order logics. The basic idea is to replace the *single global* minimality check of classical circumscription by *multiple local* minimality checks at all domain elements and their immediate neighborhood. This opens the way to use algorithmic methods (like the *mosaic technique* and *integer programming*) to obtain positive decidability results. Pointwise circumscription is a sound approximation of classic circumscription: if an ontology entails a fact under pointwise circumscription, then the entailment also holds under classic circumscription. In general, models accepted as *preferred* (or minimal) under pointwise circumscription do *not always* correspond to *preferred* models according to (classical) circumscription. It is the case of structures containing cycles: circumscription, applying minimization to all predicates globally and at once, is able to minimize all predicates involved in a cyclic dependency, while pointwise circumscription, in its local fashion, cannot.

We studied the computational complexity of reasoning under pointwise circumscription. Specifically, we considered standard DL reasoning problems (concept satisfiability, concept subsumption, and entailment of assertions) for ontologies expressed in (fragments of) the very expressive DL $\mathcal{ALCIO}$ under pointwise circumscription. We achieved decidability of reasoning in settings that are undecidable under classical circumscription. Interestingly, the computational complexity of reasoning in $\mathcal{ALCIO}$ considerably differs if quantifiers are allowed to be nested or not.

2 Overview of the Main Contributions

We call $\mathcal{ALCIO}_{d\leq 1}$ the fragment of $\mathcal{ALCIO}$ in which only TBoxes of modal depth at most 1 are allowed. The latter restriction still yields a large class of ontologies that are relevant in practice. We showed that $\mathcal{ALCIO}_{d\leq 1}$ under pointwise circumscription loses the finite model property.

We proved that reasoning in circumscribed $\mathcal{ALCIO}_{d\leq 1}$ is decidable under pointwise circumscription. We allow to minimize or fix roles, while concepts can also vary. Under

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classical circumscription role minimization leads to undecidability already in $\mathcal{ALC}$.

Main Contribution 1. *Concept satisfiability w.r.t. circumscribed KBs in $\mathcal{ALCIO}_{d\leq 1}$ is* NEXPTIME*-complete under pointwise circumscription, with roles minimized or fixed.*

The upper bound is obtained by a sophisticated reduction to integer programming. We show that the existence of a pointwise minimal model can be reduced to checking the existence of a finite family of 'fragments' of models that can be 'assembled' into a model. Following Gogacz et al.(2020a; 2020b), we encode a system of linear inequalities, where each variable corresponds to a minimal fragment of a model. A solution of the system tells how many copies of each fragment we should take to guarantee that a pointwise minimal model is built.

Our initial algorithm for concept satisfiability under pointwise circumscription is presented for concept names only. To lift it to arbitrary concept satisfiability, circumscribed KBs in $\mathcal{ALCIO}_{d\leq 1}$ are then extended with *constraints*. A constraint is a pair (C, D) of concepts in $\mathcal{ALCIO}_{d\leq 1}$ whose intuitive meaning is "if C holds, then D must hold too". An interpretation model satisfies a constraint (C, D) if all instances of C are also instances of D. Although constraints are conceptually simple, extending circumscribed KBs with constraints adds a further level of expressiveness. In a nutshell, constraints refine the set of all minimal models, filtering out those not satisfying them. We use constraints for checking the satisfiability of a complex concept w.r.t. pointwise circumscribed KBs. The latter can be reduced to checking the satisfiability of concept names w.r.t. pointwise circumscribed KBs equipped with a set of constraints, whose satisfaction ensures the satisfiability of the complex concept.

The lower bound is obtained by reducing the exponential grid tiling problem to concept satisfiability w.r.t. circumscribed KBs in $\mathcal{ALCI}$. Interestingly, if one restricts to reasoning in pointwise circumscribed $\mathcal{ALCI}_{d\leq 1}$ without ABoxes, the complexity of reasoning drops and concept satisfiability is EXPTIME-complete.

In sharp contrast with the decidability result for $\mathcal{ALCIO}_{d\leq 1}$, if no bound on the modal depth is assumed and roles are minimized, reasoning in $\mathcal{ALCIO}$ under pointwise circumscription turns out to be undecidable.

Main Contribution 2. *Reasoning w.r.t. general TBoxes in pointwise circumscribed $\mathcal{ALCIO}$ is undecidable.*

We proved the above result with a reduction from the domino problem, encoding a circumscribed KB in $\mathcal{ALCIO}$ with modal depth 8. The reduction relies on the known *spy-point* technique and does not easily carry over to $\mathcal{ALCI}$.

3 Future Research

There are several open directions for future work. Among them, we left open the problem of establishing if reasoning w.r.t. general TBoxes in $\mathcal{ALCI}$ is decidable. As mentioned, we assumed that circumscription patterns do not contain varying roles. A natural next step is to study the computational impact of varying roles. We believe that varying roles do not increase the complexity for circumscribed $\mathcal{ALCIO}_{d\leq 1}$. We are interested in studying syntactic restrictions on ontologies—such as *acyclicity* notions—to ensure that pointwise and global circumscription coincide and hence transfer some of our results to the setting of classic circumscription. In many knowledge representation examples, the two semantics coincide and in order to stress the difference between the two one needs to resort to artificial examples. This gives us hope that, in practice, it will rarely be relevant that in our semantics minimization is weaker than in global circumscription. We plan to study rewritings of pointwise circumscribed ontologies into ontologies under the standard semantics, thus enabling the reuse of existing efficient reasoners. Another promising direction is the study of the expressive power of circumscribed KBs equipped with constraints. These appear to be interesting in their own right as an additional tool for flexible yet computationally manageable non-monotonic reasoning in DLs.

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